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Brans–Dicke Theory and the Duration of the Early Universe

Marcelo Samuel Berman[1,2]

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Though it is widely accepted that the duration of the early universe is about 500,000 years, it is pointed out that in Brans–Dicke theory, with some similar hypotheses to those usually accepted in general relativity models, this duration can be only 6 years.

It is usually accepted that the duration of the early universe was 1 million years. This estimate is made by considering that, for matter,

$$\rho R^3 = \text{const} \tag{1}$$

$$R \propto t^{2/3} \tag{2}$$

$$p \cong 0 \tag{3}$$

while, from the Stefan–Boltzmann law,

$$\rho_R(T = 2.7\text{ K}) = 4.5 \times 10^{-31}\text{ kg/m}^3 \tag{4}$$

and

$$\rho_R R^4 = \text{const} \tag{5}$$

$$R \propto t^{1/2} \tag{6}$$

$$p = \frac{\rho}{3} \tag{7}$$

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[1] Department of Astronomy, University of Florida, 211 SSRB, Gainesville, Florida 32611.

[2] Permanent address: Department of Exact Sciences, Faculdade de Filosofia, Ciencias e Letras da FURJ, 89200 Joinville, SC, Brazil.

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Here, ρ stands for rest energy density, p for cosmic pressure, and R is the scale factor in the Robertson-Walker metric. It is easy to see that the condition

$$\rho_R = \rho \qquad \text{at} \quad t = t_\varepsilon \tag{8}$$

entails

$$\frac{R_\varepsilon}{R_0} \approx \frac{10^{-31}}{10^{-28}} = 10^{-3} \tag{9}$$

where R_0 is the present scale-factor value, and R_ε is its value when the radiation phase stopped being dominant ("duration of the early universe"), and we supposed the present density to be of the order of 10^{-28} kg/m^3.

Corresponding to (9), we can find, from relation (2),

$$t_\varepsilon \approx 5 \times 10^5 \text{ years} \tag{10}$$

We used the age of the universe given by

$$t_0 = \tfrac{2}{3}(H_0^{-1}) \cong 12 \times 10^9 \text{ years} \tag{11}$$

If we define $\Omega = \rho_0 / \rho_{0\text{crit}}$, where $\rho_{0\text{crit}}$ is the present critical density of the general relativistic theory, we find, instead of (10),

$$t_\varepsilon \cong 1.2\, \Omega^{-3/2} \times 10^4 \text{ years} \tag{11$'$}$$

How would the above calculations change if instead of general relativistic equations we worked with Brans-Dicke theory? To answer this question, we first make an assumption, coherent with relations (2) and (6): our models of the different phases of the universe should be of the constant-deceleration-parameter type, i.e.,

$$q = -\frac{\ddot{R} R}{\dot{R}^2} = \text{const} \tag{12}$$

If we write

$$q = m - 1 \tag{13}$$

it can be shown (Berman, 1983; Berman and Gomide, 1988) that

$$R = (mDt)^{1/m} \tag{14}$$

$$H = \frac{1}{mt} \tag{15}$$

Berman (1990) has shown that a solution for the present phase in Brans-Dicke theory is given by $m = 3$, so that

$$R \propto t^{1/3} \tag{16}$$

for the present phase, instead of $m = \frac{3}{2}$ (as in general relativity). Then, we would still have (see Berman, 1990), as valid equations, relations (1) and (5), while for the radiation phase, we would have

$$R \propto t \tag{17}$$

Relation (17) has, as a disadvantage, that we imply a negative coupling constant w. Anyway, we shall not make use of (17) in the following calculations.

We again have

$$\frac{R_\varepsilon}{R_0} \approx 10^{-3} \tag{18}$$

and then, by (16), we find, with $t_0^{(\mathrm{BD})} = 6 \times 10^9$ years, as given by (15), with $m = 3$,

$$t_\varepsilon^{(\mathrm{BD})} \approx 6 \text{ years} \tag{19}$$

In terms of Ω, we would have

$$t_\varepsilon^{(\mathrm{BD})} = \frac{6 \times 10^{-3}}{\Omega^3} \text{ years} \tag{20}$$

As one may check, the difference in (19) when compared with result (10) makes us guess, tentatively, that it is very difficult to be sure that some accepted results in cosmology are really true. The fact that Brans-Dicke solution may differ radically from the general relativity solution for the same physical problem was stressed by Cervero and Estevez (1983; see references therein). Note that, if we accept the result (19), we would not have any problem with nuclear synthesis (see, for instance, Fowler, 1989).

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